

Patch-Based Image Denoising in the Wavelet Domain

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Abstract - Patch-based methods used in digital image processing fields are generally able to produce effective results. Although these approaches use easier structures to achieve better visual quality in digital image restoration compared with other methods, research is still going on in the field. In this study, a better noise reduction approach is presented using a patch-based algorithm in the wavelet domain. We implemented an isotropic patch-based smoothing method similar to the non-local means algorithm in the wavelet domain. Results of the method show that additive Gaussian noise is efficiently removed from grayscale images while image edges and textures remain unblurred.

Keywords: Patch-Based Denoising, Redundant Wavelet Transform, Isotropic Smoothing, Additive Gaussian Noise.

1 Introduction

To obtain visually plausible results and to improve the peak signal-to-noise ratio (PSNR), edges and textures should be preserved in digital image denoising. For this purpose two remarkable approaches have been suggested recently, which are wavelet-based and patch-based noise reduction [1-2, 4, 7-8]. Classical image denoising methods such as partial differential equation (PDE) based anisotropic diffusion [3] and fuzzy-based smoothing methods are not able to conserve image structures. If these methods are not used together as a hybrid method, the results include more blurring effects [2, 4, 5, 6, 8, 11].

Wavelet-based noise reduction, using thresholding or shrinkage operation, has been at a premium due to the fact that it produces very effective results. Pizurica *et al.* [1] presented a probabilistic shrinkage method in the wavelet domain to reduce noise in digital images. Owing to its greater complexity, Schulte *et al.* [2] proposed a fuzzy shrinkage algorithm, which has less execution time. On the other hand, Buades *et al.* [7, 10] introduced the non-local means noise removal approach, which is a patch-based method. Due to its efficiency, Tschumperle *et al.* [4, 9] adapted the method to be used in patch-space with PDE. Zhong *et al.* [8] presented wavelet based multi-scale anisotropic smoothing algorithm. Louchet *et al.* [11] presented a denoising approach, Total Variation (TV)-means, that is based on local TV-filtering and the non-local means method. However, most of the previously mentioned frameworks can be improved by using composite approaches.

We presented a hybrid method to remove additive Gaussian noise from grayscale images without degenerating the image structure, which results in superior results.

2 Image Denoising

Let $I_{noisy}: \Omega \rightarrow \mathbb{R}$ be a noisy 2 – dimensional grayscale image to be denoised. The noise can be seen as high frequency variations n with low magnitude in pixels of the original image [4-6]:

$$I_{noisy} = I + n \quad (1)$$

where I is an original image.

The denoising operation of I_{noisy} is to minimize the variations in the noisy image. Related proposed approaches are explained briefly in the following sections.

2.1 Wavelet-based Denoising

Wavelet-based approaches used in noise reduction generally contain the following steps [2]:

- a) Obtaining wavelet coefficients
- b) Removing the noise from the noisy wavelet coefficients
- c) Getting the reconstructed image by means of the inverse wavelet transform

Since the wavelet transform is linear, additive noise in the spatial domain corresponds to additive in the wavelet domain [2, 8].

Let $I^{w,s,d}$ and $I^{y,s,d}$ indicate the noisy and noiseless wavelet coefficients of scale s and orientation d so that the additive noise is formulized in the wavelet domain as follows [2, 8]:

$$I^{w,s,d} = I^{y,s,d} + I^{n,s,d} \quad (2)$$

where $I^{n,s,d}$ is the noise term.

The most important step in the wavelet-based noise reduction approaches is to shrink the wavelet coefficients. If they contain noise, they are reduced to insignificant values. Otherwise, they are reduced less. The classical shrinkage approach is the application of simple thresholding nonlinearities to the empirical wavelet coefficients. If the

absolute value of the coefficient is below the threshold T , it is reduced to zero; if not, it is kept or modified [2].

If a certain wavelet coefficient and its neighboring coefficients are small enough, it is almost noisy and is set to zero. Coefficients above a predefined threshold contain the most important image structures and are not reduced. But they contain both noise and image data. An efficient threshold is generally chosen so that approximate coefficients below and above the threshold are noise and image data respectively. In this case, using fuzzy-based approaches is more efficient as a soft-threshold method [2].

2.2 Patch-based Denoising

Let a patch $\mathcal{P}_{(x,y)}^I$, with center $\mathbf{x} = (x, y)$, define the set of all image values and a spatially discretized local $p \times p$ neighborhood of I [4]. Here, the spatial discretization step of the patch is set to 1. The dimension p is chosen as odd, i.e., $p = 2q + 1, q \in \mathbb{N}$. The patch $\mathcal{P}_{(x,y)}^I$ can be written in a $p^2 -$ dimensional vector form as follows (see Figure 1):

$$\mathcal{P}_{(x,y)}^I = (I_{(x-p,y-q)}, \dots, I_{(x+p,y+q)}) \quad (3)$$

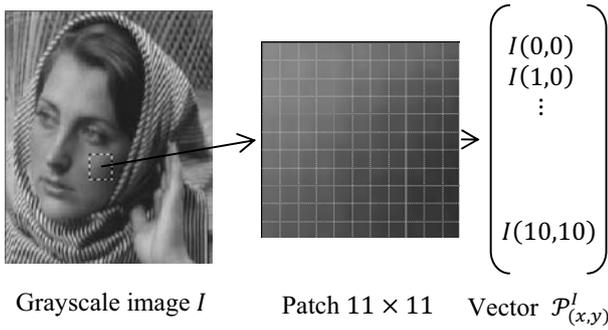


Figure 1. Defining an image patch $\mathcal{P}_{(x,y)}^I$.

And let the $(p^2 + 2) -$ dimensional patch-space be defined as $\Gamma = \Omega \times \mathbb{R}^{p^2}$. Each point \mathbf{p} of Γ is a high-dimensional vector which contains both information of the (x, y) coordinates in Ω and of all values of the $p \times p$ patch \mathcal{P} in \mathbb{R}^{p^2} . Let $\mathbf{p} = (x, y, \mathcal{P}_{(x,y)}^I)$ denote points in Γ . These locations certainly belong to existing patches in I . The Euclidian distance between $\mathbf{p}_1, \mathbf{p}_2 \in \Gamma$ measures a spatial and dissimilarity between corresponding patches:

$d(\mathbf{p}_1, \mathbf{p}_2) = \sqrt{\|\mathbf{x}_1 - \mathbf{x}_2\|^2 + c^2 SSD(\mathcal{P}_1, \mathcal{P}_2)}$. Here, SSD stands for the sum of squared differences and $c > 0$ stabilizes importance of spatial and intensity features [4] (see Figure 2). Let the function \tilde{I} be defined in Γ and $\tilde{I}_{(\mathbf{p})}$ as having a value for these located patches: $\tilde{I}: \Gamma \rightarrow \mathbb{R}^{p^2+1}$, and $\forall \mathbf{p} \in \Gamma$,

$$\tilde{I}_{(\mathbf{p})} = \begin{cases} (I_{(x,y)}, 1) & \text{if } \mathbf{p} = (x, y, \mathcal{P}_{(x,y)}^I) \\ \vec{0} & \text{elsewhere} \end{cases} \quad (4)$$

It is defined operation F that $\tilde{I} = F(I)$ figures out a patch-based denotation of I . In (4), the value space of \tilde{I} owns

an additional component whose value is 1 at the located patches of I . This value is taken into account during inverse transform F , i.e., getting back I from \tilde{I} .

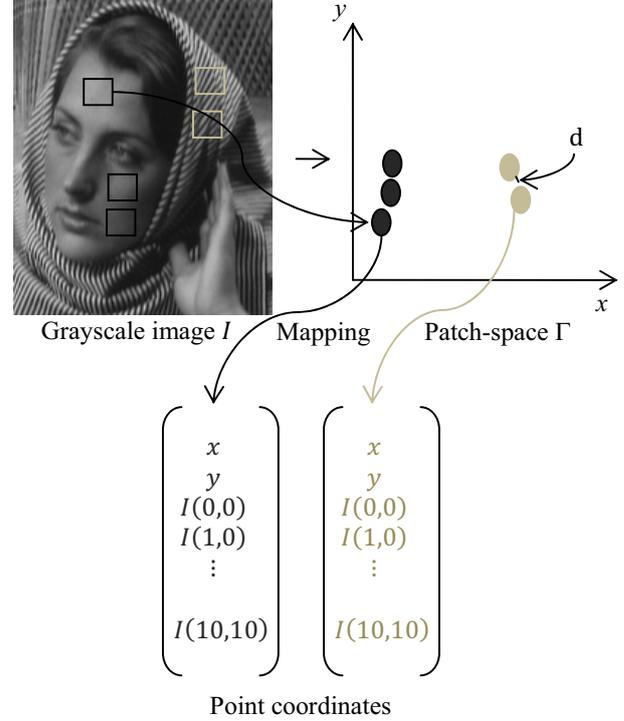


Figure 2. Mapping grayscale image I into patch-space Γ .

On account of the high dimensionality of Γ , to invert the patch transform $\tilde{I} = F(I)$, Tschumperle *et al.* [4] defined a back-projection method based on two steps: First, the most significant patch $\mathcal{P}_{sig(x,y)}^I$ in Γ is gotten back for corresponding location (x, y) in Ω . The solution with the maximum projective weight is as follows:

$$\mathcal{P}_{sig(x,y)}^I = \operatorname{argmax}_{\mathbf{q} \in \mathbb{R}^{p^2}} I_{p^2+1}(x, y, \mathbf{q}) \quad (5)$$

It can be found out that the significant patches $\mathcal{P}_{sig(x,y)}^I$ reside in the same locations of $\mathcal{P}_{(x,y)}^I$ as the original ones by using the patch denotation \tilde{I} of I , even though the pixel values of these most significant patches may have been modified. Secondly, the back-projected image \tilde{I} is reconstructed by combining the most significant patches together. For this, Tschumperle *et al.* [4] used the simplest possible strategy, i.e., copying the normalized center pixel of each $\mathcal{P}_{sig(x,y)}^I$ at its corresponding location (x, y) :

$$\forall (x, y) \in \Omega, \tilde{I}_{(x,y)} = \frac{\tilde{I}_{p^2+p^2+1}(x, y, \mathcal{P}_{sig(x,y)}^I)}{\tilde{I}_{p^2+1}(x, y, \mathcal{P}_{sig(x,y)}^I)} \quad (6)$$

Suppose that a grayscale image I_{noisy} is corrupted with additive Gaussian noise and its patch transform is denoted as \tilde{I}_{noisy} . A patch-based minimizing flow approach, which is defined in [4], regularizes \tilde{I}_{noisy} rather than processes I_{noisy} directly. The energy E , which is known as an isotropic regularization functional and was extended to the high-dimensional space Γ in [4], is minimized as follows:

$$E(\tilde{I}) = \int_{\Gamma} \|\nabla \tilde{I}(\mathbf{p})\|^2 d\mathbf{p} \quad (7)$$

The necessary condition to solve (7) is given by the Euler-Lagrange equations, which must be confirmed by \tilde{I} to reach a minimum of $E(\tilde{I})$ and by the classic iterative method, which is the gradient descent. It is also called the heat flow (diffusion) equation which is achieved, in this case, on the high-dimensional patch space Γ :

$$\begin{cases} \tilde{I}_{[t=0]} = \tilde{I}_{noisy} \\ \frac{\partial \tilde{I}}{\partial t} = \Delta \tilde{I} \end{cases} \quad (8)$$

where Δ is the Laplace operator on Γ .

3 The Proposed Method

To reduce additive Gaussian noise, we presented a combined approach using the redundant wavelet transform (RWT) and patch-based denoising.

One of the main properties of the discrete wavelet transform is significant reduction of wavelet coefficients. This approach provides not only faster wavelet transform, but also less memory usage. Unfortunately, this property causes shift variance of the wavelet transform. In other words, minor shifts in the input data are able to bring about crucial variations in the energy distribution among coefficients at different resolution scales. For this reason, we used the RWT to overcome this difficulty, despite its drawbacks, e. g., more computation and more memory usage. There are output images for the RWT of Barbara's grayscale image with dimension 512×512 in Figure 3.

After the wavelet transform, we performed the patch-based noise reduction approach on wavelet decomposition of the input image rather than on the directly noisy one. Tschumperle *et al.* [4] presented a solution for the patch-based heat flow at a particular finite time t , which is the convolution of the initial estimate \tilde{I}_{noisy} with normalized Gaussian kernel G_{σ} and standard deviation $\sigma = \sqrt{2t}$. The convolution is performed on the high-dimensional patch space Γ :

$$\tilde{I} = \tilde{I}_{noisy} * G_{\sigma} \text{ with } \forall \mathbf{p} \in \Gamma, \quad (9)$$

$$\text{where } G_{\sigma}(\mathbf{p}) = \frac{1}{(2\pi\sigma^2)^{\frac{p^2+2}{2}}} e^{-\frac{\|\mathbf{p}\|^2}{2\sigma^2}}.$$

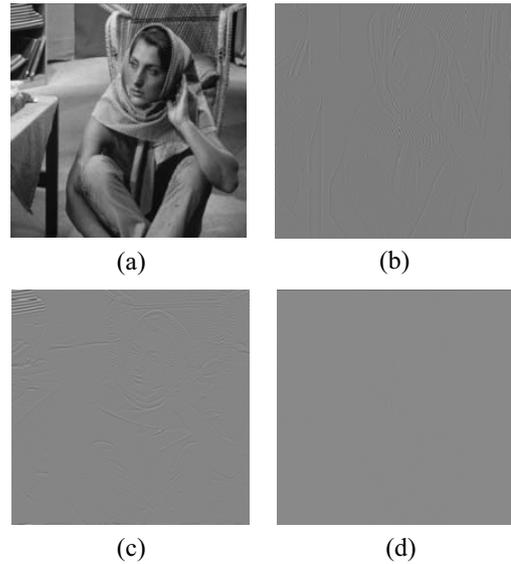


Figure 3. One-level RWT for Barbara's grayscale image: (a) base, (b) vertical, (c) horizontal and (d) diagonal bands.

Thus, the convolution, given in (9), can be rewritten in the wavelet domain as follows:

$$\begin{aligned} \forall (x, y, \mathcal{P}) \in \Gamma, \\ \tilde{I}^{y,s,d}(x, y, \mathcal{P}) = \int_{\Omega} \tilde{I}^{w,s,d} \left(\begin{matrix} p,q,\mathcal{P} \\ (x,y) \end{matrix} \right) G_{\sigma} \left(\begin{matrix} p-x,q-y,\mathcal{P} \\ (p,q) \end{matrix} \right) dpdq \end{aligned} \quad (10)$$

Actually, the non-local means framework [7] is similar to the patch-based isotropic smoothing method [4]. We employed the same patch-based approach to remove noise from the wavelet coefficients due to the fact that it achieves a very good performance, instead of the classical soft-based thresholding technique proposed in [2].

4 Experimental Results

As is seen in Figure 4.a-b and Figure 5.a-b, an artificial Gaussian noise ($\sigma = 20$) is added to Barbara and Peppers grayscale images with dimensions of 512×512 for tests. The proposed method is compared with the anisotropic smoothing [3], the patch-based approach [4, 7, 10], and the fuzzy shrinkage algorithm [2]. RWT with 'db4' wavelet decomposition and four resolution scales are used for the fuzzy shrinkage algorithm and the proposed method.

The denoised images obtained by the proposed approach, and other methods mentioned above are shown in Figure 4.c-f and Figure 5.c-f. In addition to this, PSNR between the original image and restored ones are given in these Figures. The proposed hybrid method accomplishes the best denoising performance compared with the three other approaches, according to PSNR values, as can be seen in Figure 4.c-f and Figure 5.c-f. In regards to the visual quality of the output images for these algorithms, the proposed

method achieves the best plausible visual result. For the anisotropic smoothing and the patch-based method, which are directly performed on the spatial domain, noise cannot be removed adequately. On the other hand, the wavelet-based approaches, which are the fuzzy shrinkage algorithm and the proposed method, are more effective to reduce the noise. However, the fuzzy shrinkage method occasionally degrades some image regions, e. g., Barbara's nose, whereas the proposed method reconstructs the output image almost perfectly. As a result, the proposed method presents a better performance compared with the other methods as can be seen in the denoised results.

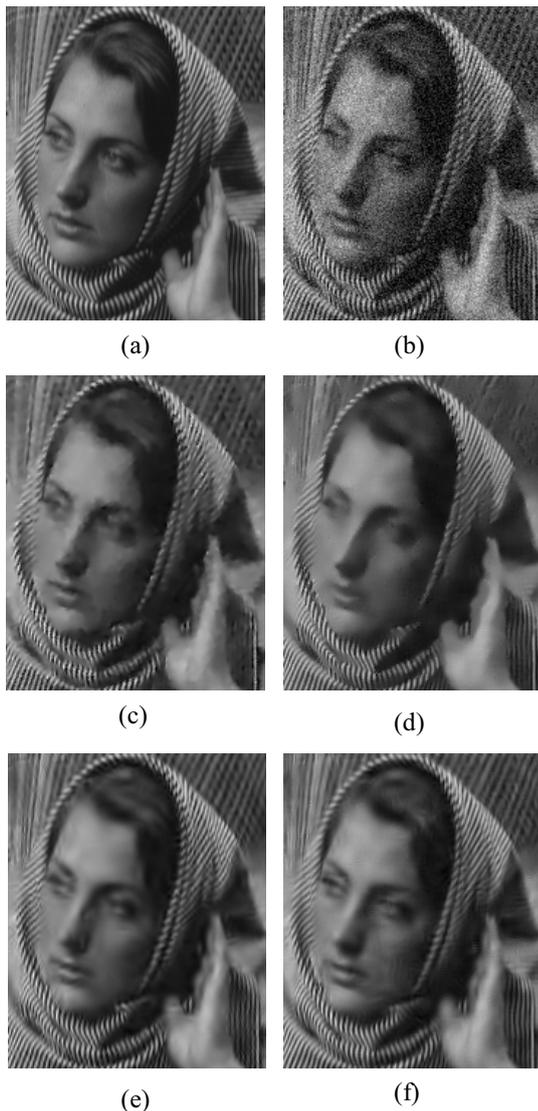


Figure 4. Close-up of Barbara images for restoration: (a) the original image (b) the noisy image artificially corrupted with additive Gaussian noise ($\sigma=20$) (c) Anisotropic PDE (PSNR=27.26 dB), (d) Patch-based (PSNR=29.15 dB), (e) Fuzzy shrinkage (PSNR=30.16 dB), (f) Proposed method (PSNR=30.63 dB).

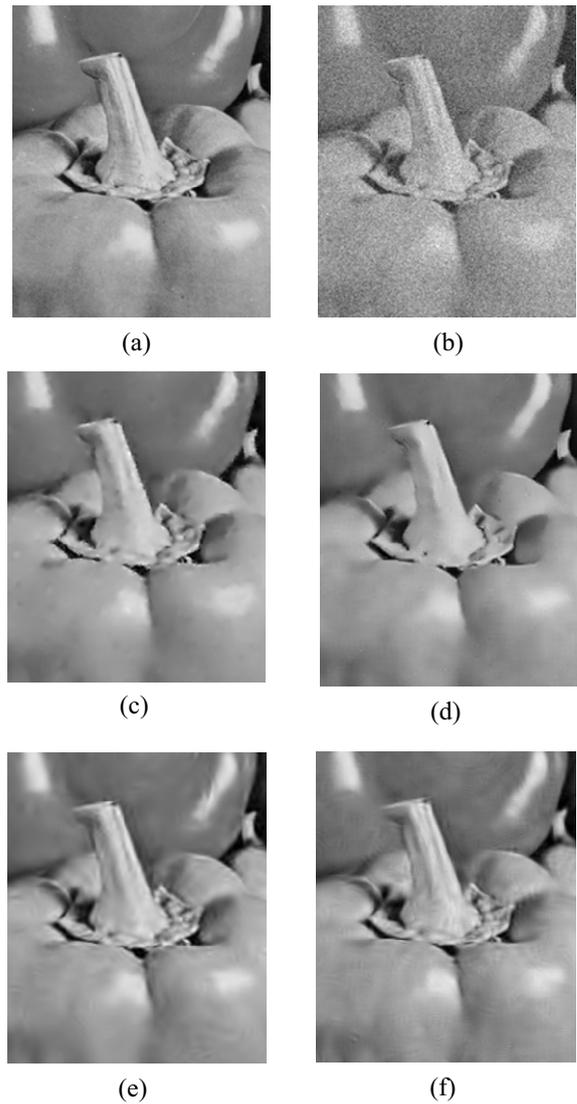


Figure 5. Close-up of Peppers images for restoration: (a) the original image (b) the noisy image artificially corrupted with additive Gaussian noise ($\sigma=20$) (c) Anisotropic PDE (PSNR=31.37 dB), (d) Patch-based (PSNR=31.50 dB), (e) Fuzzy shrinkage (PSNR=31.59 dB), (f) Proposed method (PSNR=31.69 dB).

These methods were implemented in Microsoft Visual C++ 2005. The program was run on a laptop with a Pentium 2.20 GHz processor and 2 GB RAM.

5 Conclusion

In this paper, a patch-based method using wavelet domain is suggested to remove the additive Gaussian noise from grayscale images. This method reconstructs the denoised image very well while preserving the image properties such as edges. As proved with experimental results, the performance and output of the proposed method are encouraging. The results present good visual quality and numerical measures.

The computational complexity of the proposed method, which necessitates the longest execution time compared with the other approaches, will be reduced as a future task.

6 References

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